5. LARGE-SCALE ADJUSTMENT

The aim of this final lecture is to explain how the large ocean circulation adjusts to localised forcing anomalies.

First we will review the dominant wave modes involved in this adjustment:

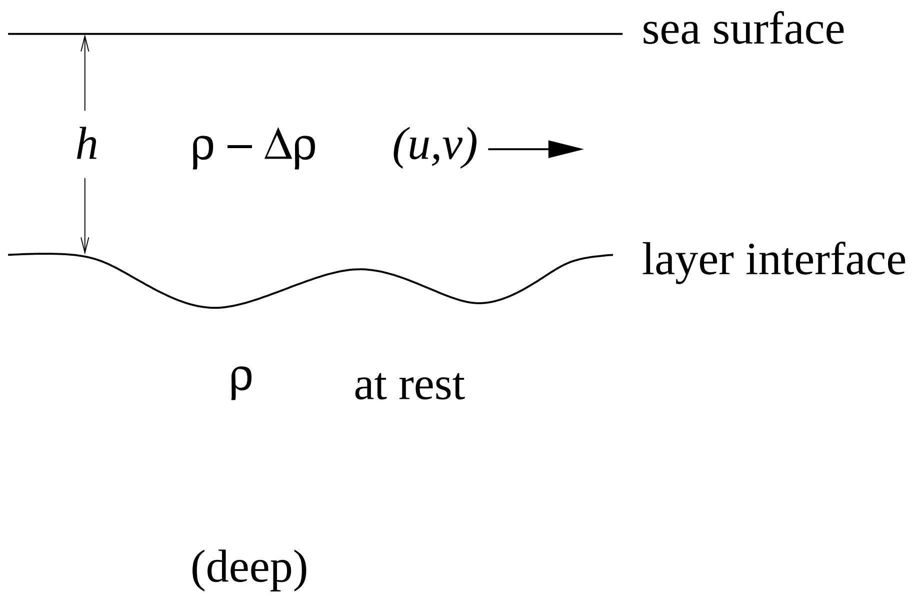
- long Rossby waves,
- Kelvin waves.

Then we will consider some recent models for the basin-scale adjustment to anomalies in deep water formation at high latitudes.

Although we will not describe them here, similar results are obtained for the response to wind forcing anomalies (see, for example, Cessi and Louazel 2001, Cessi and Otheguy 2003).

SHALLOW-WATER MODEL

We will restrict our attention to a reduced-gravity ocean consisting of one moving layer overlying a motionless abyss.



The equations of motion are:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv + g'\frac{\partial h}{\partial x} = 0, \tag{5.1}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + fu + g'\frac{\partial h}{\partial y} = 0, \tag{5.2}$$

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) = 0, \tag{5.3}$$

where

$$g' = g\,\Delta\rho/\rho_0$$

is the reduced gravity.

LONG ROSSBY WAVES

First consider the large-scale interior of an ocean basin, where Ro $\ll 1$, and the momentum equations can be approximated by geostrophic balance:

$$u = -\frac{g'}{f}\frac{\partial h}{\partial y}, \quad v = \frac{g'}{f}\frac{\partial h}{\partial x}. \tag{5.4}$$

Substituting these into the continuity equation (5.3) gives:

$$\frac{\partial h}{\partial t} - c(y)\frac{\partial h}{\partial x} = 0, \tag{5.5}$$

where

$$\begin{aligned} c(y) &= \frac{\beta g' h}{f^2} \\ &= \beta L_D^2 \end{aligned} \tag{5.6}$$

is the long Rossby wave speed, and

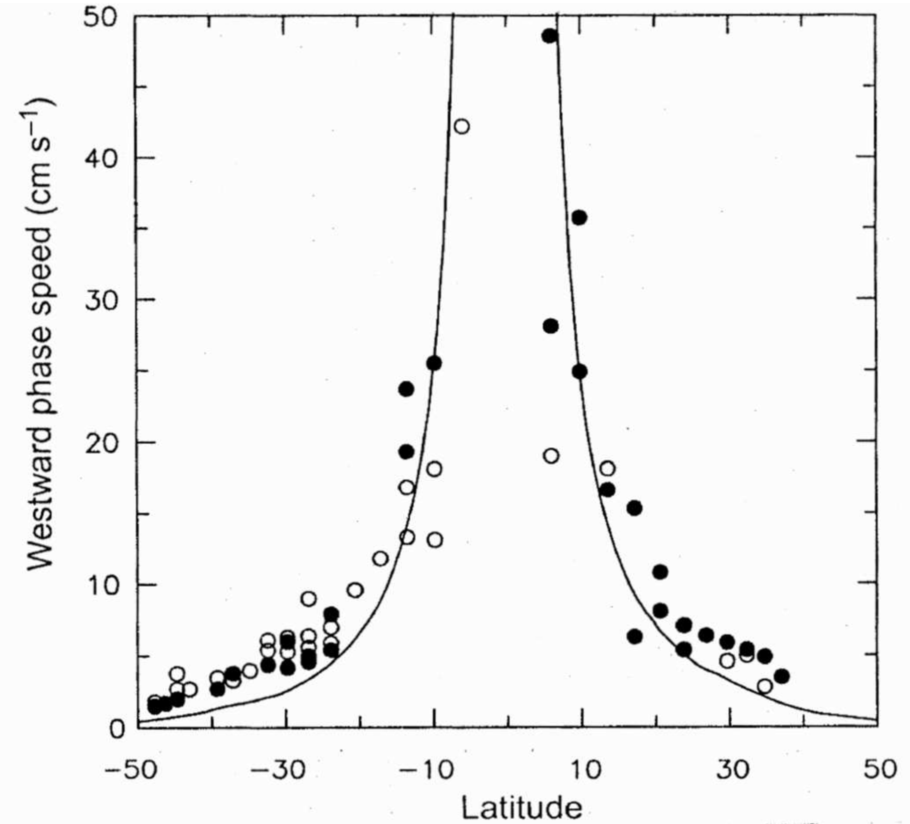
$$L_D = \frac{\sqrt{g' h}}{f} \tag{5.7}$$

is the Rossby deformation radius.

Thus *all* large-scale interior anomalies propagate westward in this model at the long Rossby wave speed.

[Note (5.6) is the long-wave limit ($\lambda \gg L_D$) of the more general Rossby wave speed; see PPH lectures.]

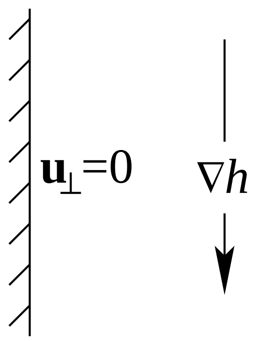
Estimated speed of westward propagating anomalies from altimeter data (● = Pacific; ○ = Atlantic). Solid line is the theoretical prediction from (5.6).



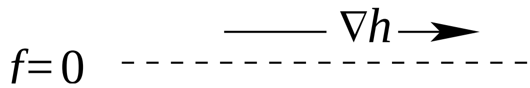
(Chelton and Schlax 1986)

KELVIN WAVES

Suppose we have a pressure gradient along a coastline or along the equator. These pressure gradients *cannot* be balanced by a Coriolis force.



Instead we obtain a *Kelvin wave.*



The simplest case is of a Kelvin wave along a north-south coastline with constant f. The vanishing of u at the coastline suggests the possibility of a solution in which $u = 0$ everywhere.

Linearising the remaining terms in (5.1-5.3) about a state of rest gives:

$$-fv + g'\frac{\partial h}{\partial x} = 0, \tag{5.8}$$

$$\frac{\partial v}{\partial t} + g'\frac{\partial h}{\partial y} = 0, \tag{5.9}$$

$$\frac{\partial h}{\partial t} + H\frac{\partial v}{\partial y} = 0, \tag{5.10}$$

where H is the mean layer thickness.

Eliminating v between (5.9) and (5.10) gives a wave equation:

$$\frac{\partial^2 h}{\partial t^2} - g'H\frac{\partial^2 h}{\partial y^2} = 0. \tag{5.11}$$

This admits two waves propagating at speeds $c = \pm\sqrt{g'H}$:

$$h = A(x)\,F\,(y - ct) + B(x)\,G\,(y + ct)\,. \tag{5.12}$$

To determine the zonal structure of the wave, eliminate v between (5.8) and (5.9) to give:

$$\frac{\partial^2 h}{\partial x \partial t} + f\frac{\partial h}{\partial y} = 0. \tag{5.13}$$

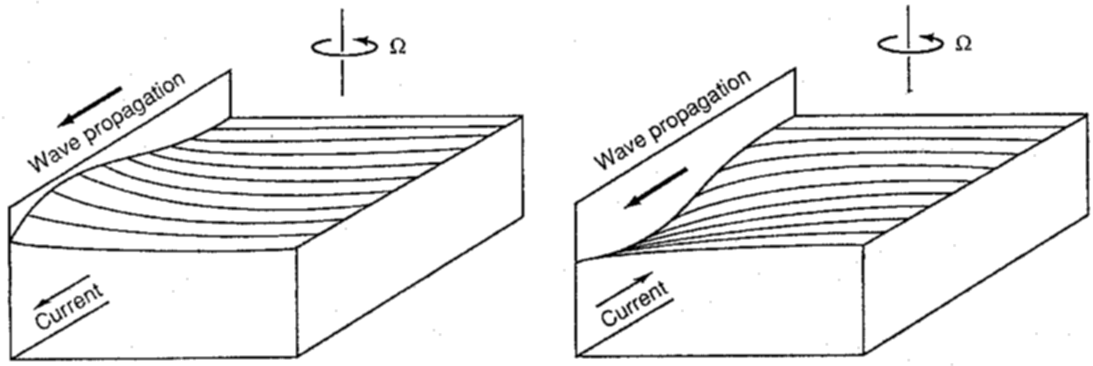
Substituting the above solution gives:

$$\frac{dA}{dx} = \frac{A}{L_D}, \quad \frac{dB}{dx} = -\frac{B}{L_D}, \tag{5.14}$$

and thus:

$$A = A_0 e^{x/L_D}, \quad B = B_0 e^{-x/L_D}. \tag{5.15}$$

Only the wave that decays away from the boundary is physical $\Rightarrow$ the wave travels with the coast to its right in the Northern Hemisphere, and with the coast to its left in the Southern Hemisphere.



(from Cushman Roisin 1994)

Essentially an internal gravity wave in the direction $||$ to coast, but in geostrophic balance in direction $\perp$ to coast.

Speed: $g' \sim 10^{-2}\mathrm{m\,s^{-2}}$, $H \sim 400\,\mathrm{m} \Rightarrow c \sim 2\,\mathrm{m\,s^{-1}}$

$\Rightarrow$ few months to propagate from high latitudes to equator.

Decay scale: $L_D \sim 30\,\mathrm{km}$ (at midlatitudes).

Similarly find *equatorial Kelvin waves* along the equator —

like two coastal Kelvin waves leaning against each other.

KELVIN WAVES AND THE β-EFFECT

When the Coriolis parameter varies with latitude, classical Kelvin wave solutions are obtained only at high frequencies (see Clarke and Shi 1991).

For periods greater than about a month, we find:

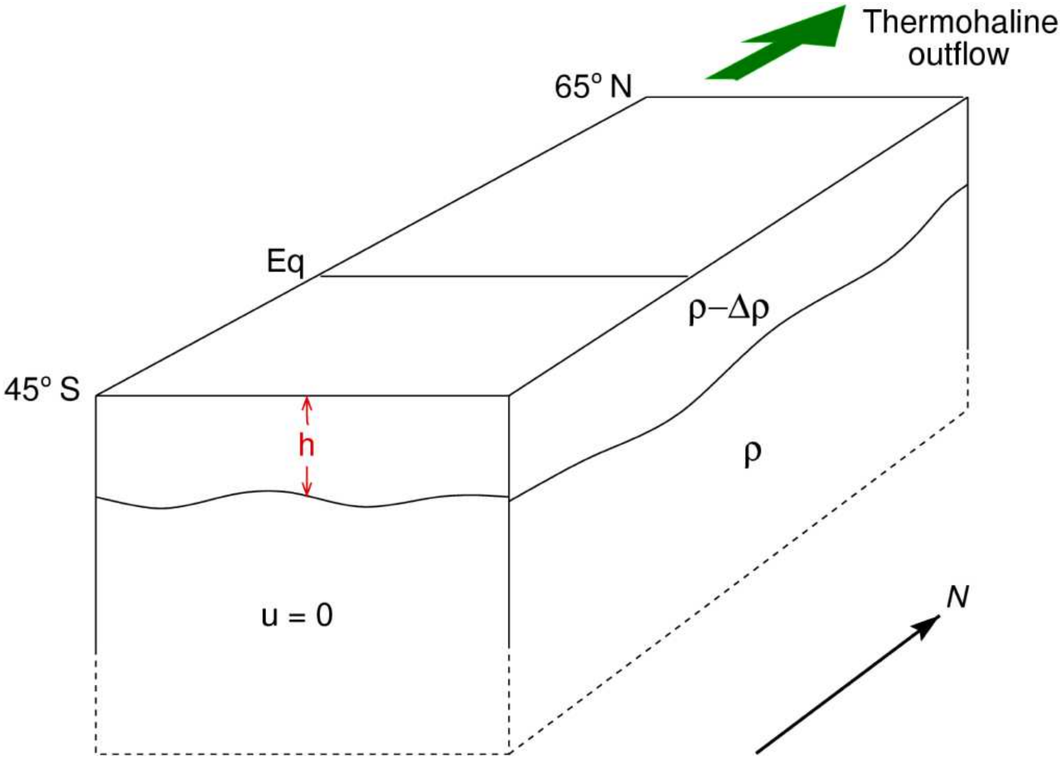
- the pressure anomalies are no longer trapped at the boundary, but radiate as short/long Rossby waves from the western/eastern boundaries respectively.
- the propagation speed *along* the boundary is frictionally controlled ($\Rightarrow$ model dependent!)
- as a western boundary wave propagates equatorward, the pressure anomaly decays but the meridional transport anomaly is constant; but as an eastern boundary wave propagates poleward, the pressure anomaly is constant and the meridional transport anomaly decays.

At very large periods, the western boundary solution is replaced by an oscillating frictional boundary current (consistent with the steady-state limit).

ADJUSTMENT OF THE MERIDIONAL OVERTURNING CIRCULATION

- Over what time-scales does the ocean respond to changes in deep-water formation at high latitudes?
- How localised is variability on different time-scales?
- Can the adjustment be described using a simple model?

Reduced-gravity model of upper limb of Atlantic overturning circulation (Johnson and Marshall 2002a):

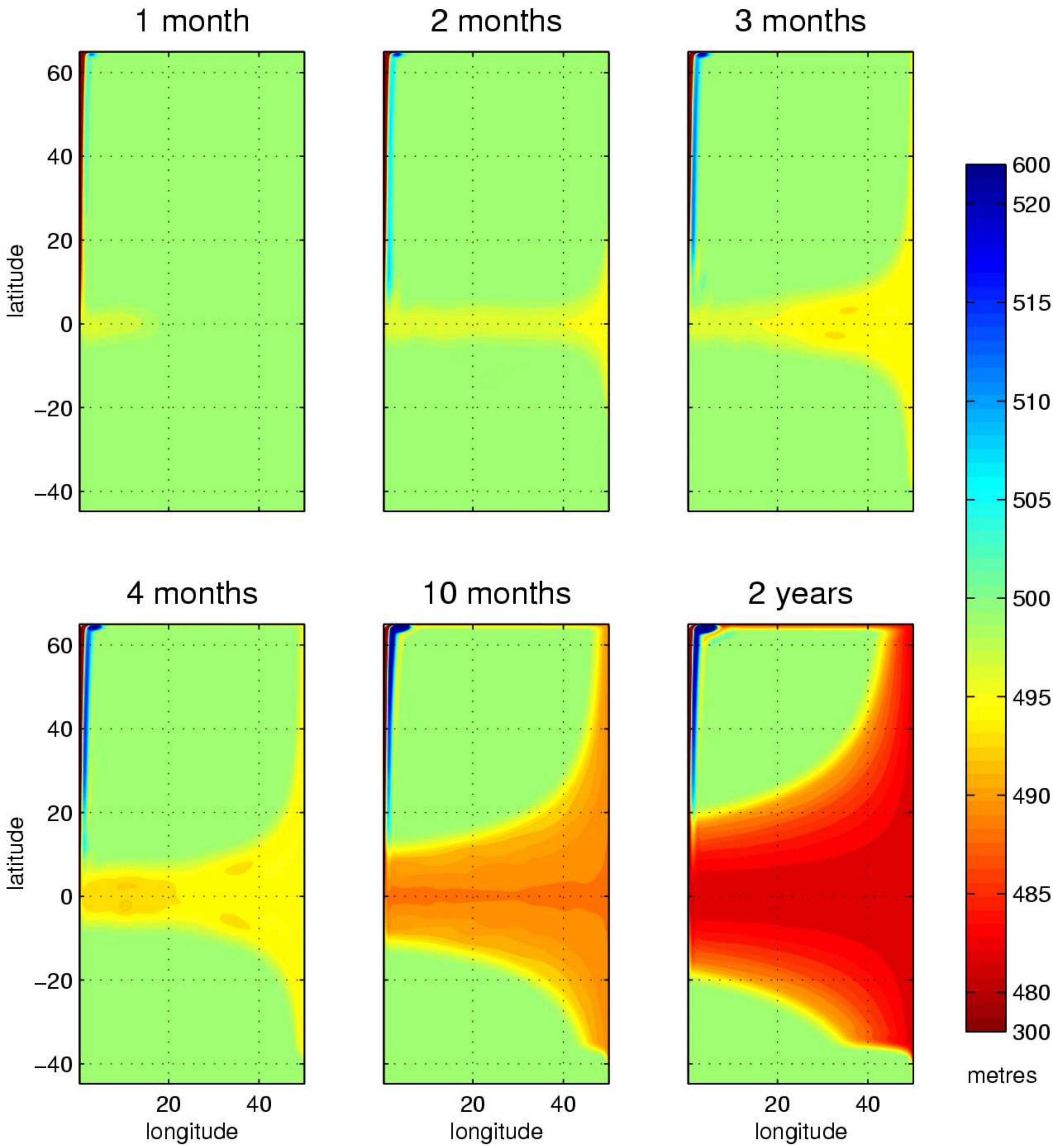


- Dynamic upper layer (initially $h = 500\,\mathrm{m}$) overlying motionless abyss
- Domain from 45°S to 65°N, and 50° wide
- Prescribed outflow at northern boundary
- Layer thickness relaxed to uniform value (500 m) at southern margin (45°S - 35°S)

IMPULSIVE CHANGE IN DEEP WATER FORMATION

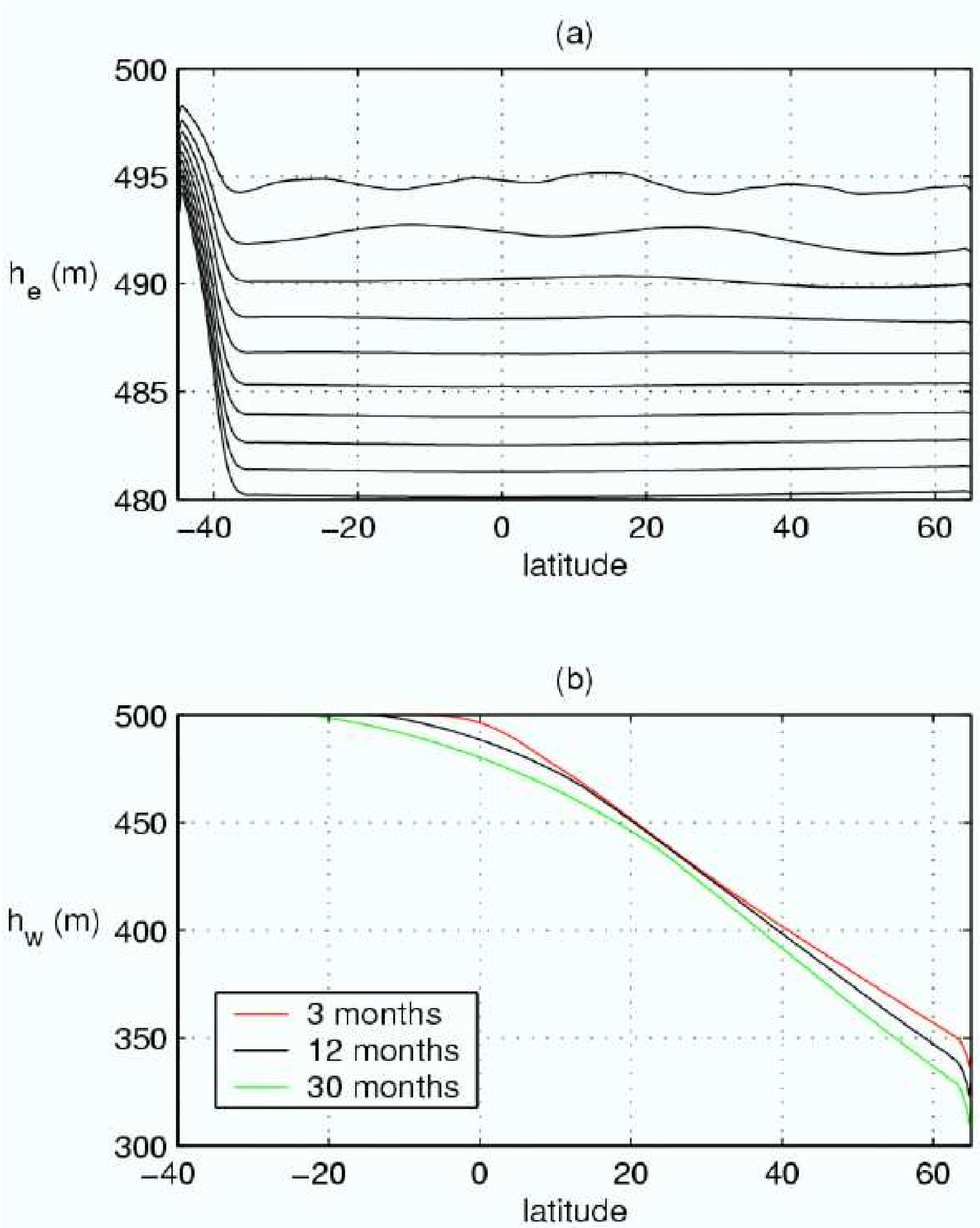
Northern outflow turned on at $t = 0$ (inverse shutdown!)

Layer thickness (m):

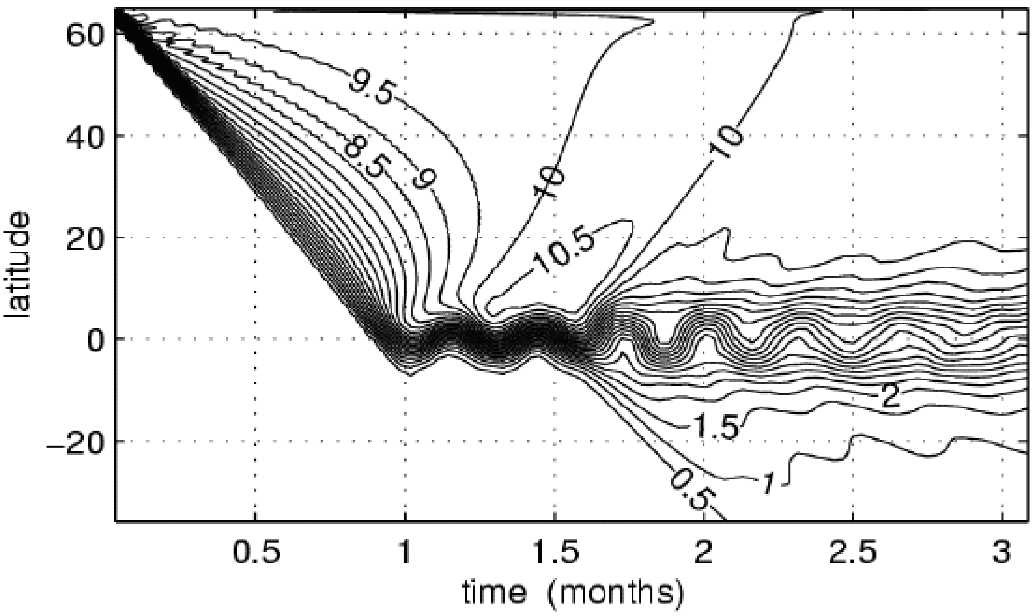


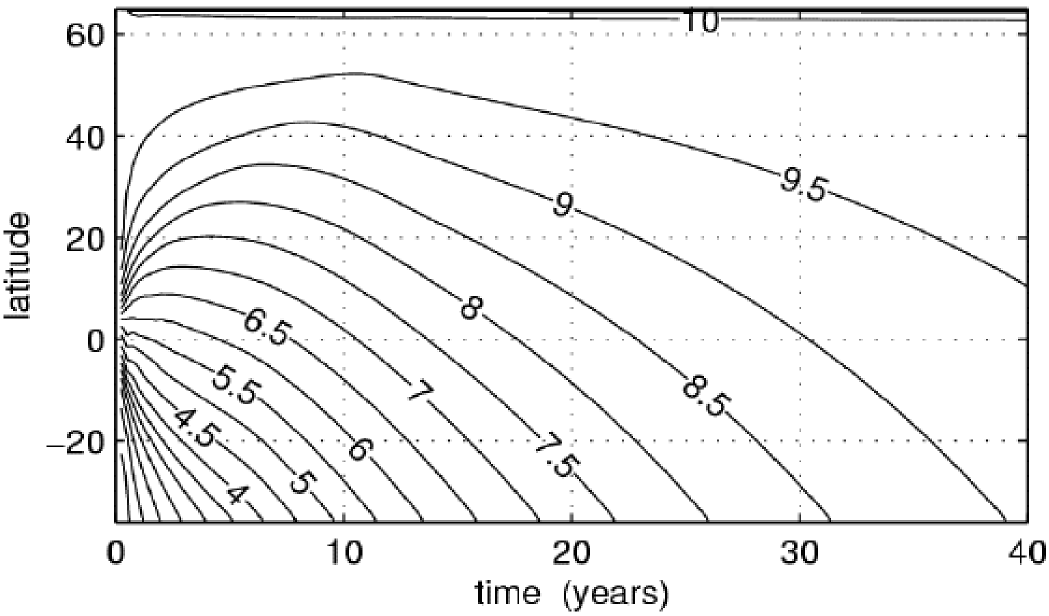
(cf. Wajsowicz and Gill 1986, Kawase 1987, Yang 1999)

Layer thickness (m) along eastern and western boundaries:

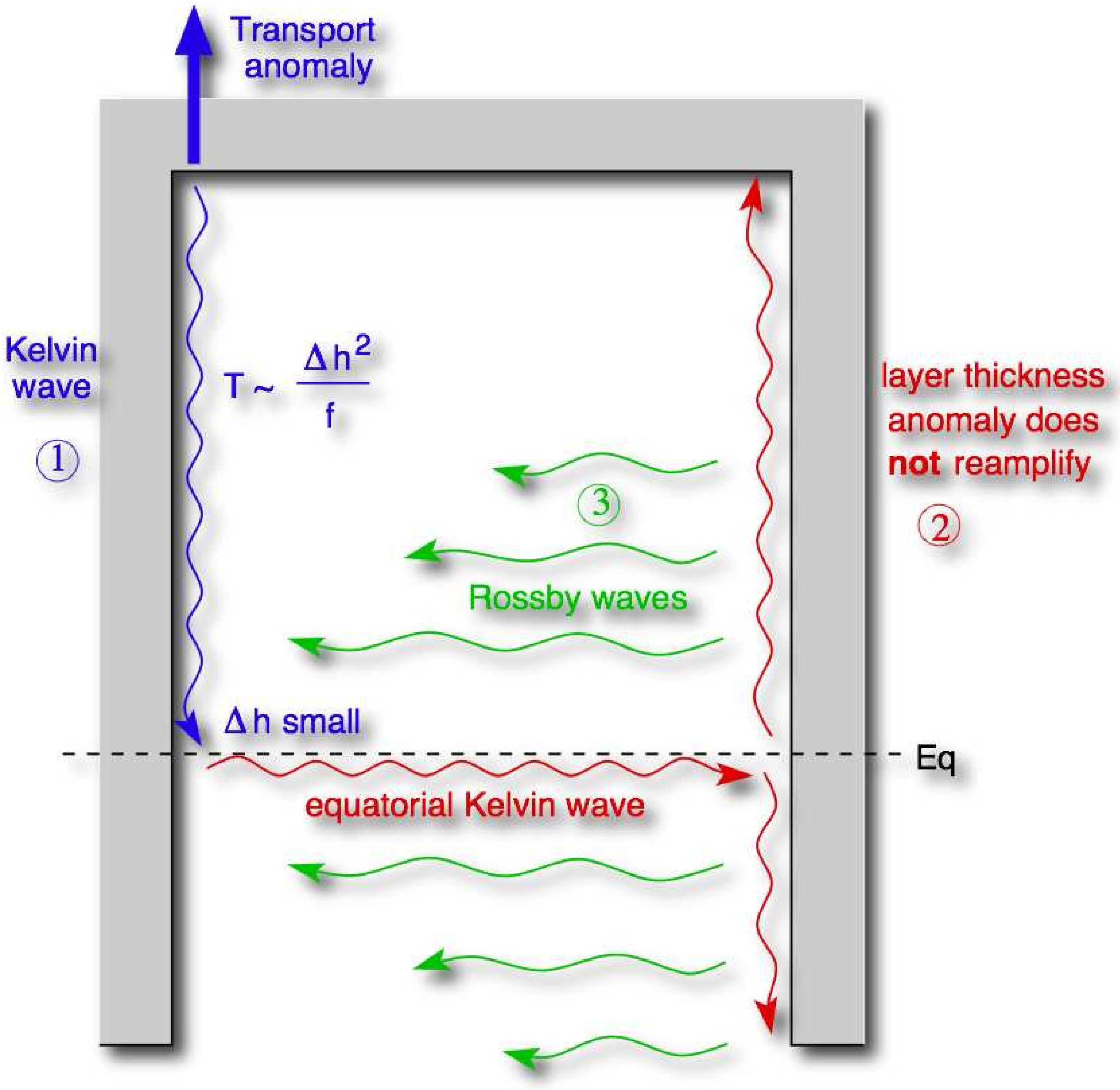


Northward transport (Sv):





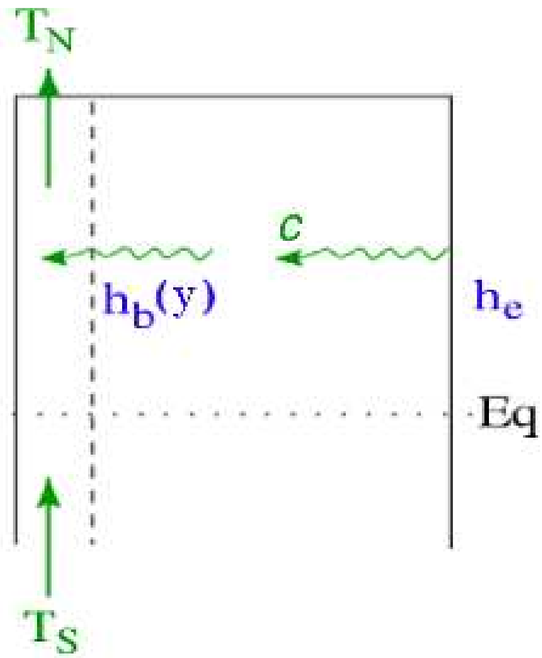
Summary of the adjustment mechanisms:



Equatorial buffer ⇒ basin interior/southern hemisphere "sees" only a small fraction of the original forcing anomaly.

SIMPLE THEORY FOR THE ADJUSTMENT

Assumptions:



- Kelvin waves infinitely fast;
- h_e uniform;
- western boundary layer narrow;
- Ro $\ll$ 1 in basin interior;
- linear.

(i) Basin volume balance:

$$\frac{\partial}{\partial t} \iint_{\text{basin}} h\, dx\, dy = T_S - T_N. \tag{5.16}$$

(ii) Interior volume budget:

Rossby wave equation:

$$\frac{\partial h}{\partial t} - c(y)\frac{\partial h}{\partial x} = 0, \tag{5.17}$$

where $c(y)$ is the long Rossby wave speed in (5.6).

$$\Rightarrow \frac{\partial}{\partial t} \iint_{\text{interior}} h\, dx\, dy = \int_{y_S}^{y_N} c(y) \left\{ h_e(t) - h_b(y,t) \right\} dy, \tag{5.18}$$

where $h_b(y,t) = h_e(t - L/c)$.

Delay equation

Finally equating (5.16) and (5.18) gives:

$$\int_{y_S}^{y_N} c(y) \left\{ h_e(t) - h_e\left(t - \frac{L}{c(y)}\right) \right\} dy = T_S - T_N. \qquad (5.19)$$

Only one unknown: $h_e(t)$

Solution for the remaining variables

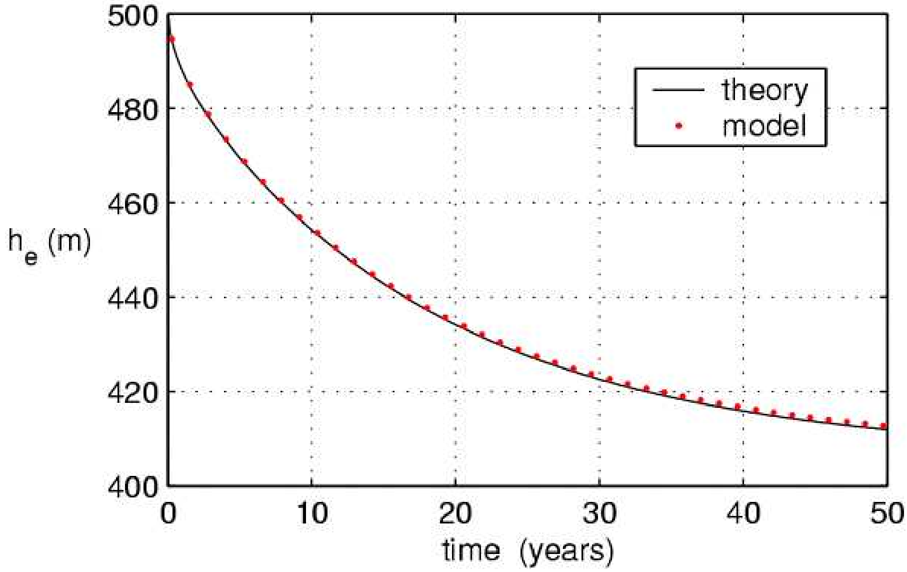
From $h_e(t)$, the layer thickness in the interior follows from the Rossby wave equation:

$$h(x, y, t) = h_e\left(t - \frac{x_e - x}{c(y)}\right). \qquad (5.20)$$

The net northward transport can be found as a function of latitude by equating (5.16) and (5.18) with the integration now carried out between an arbitrary latitude, y, and the northern boundary, y_N. This gives:

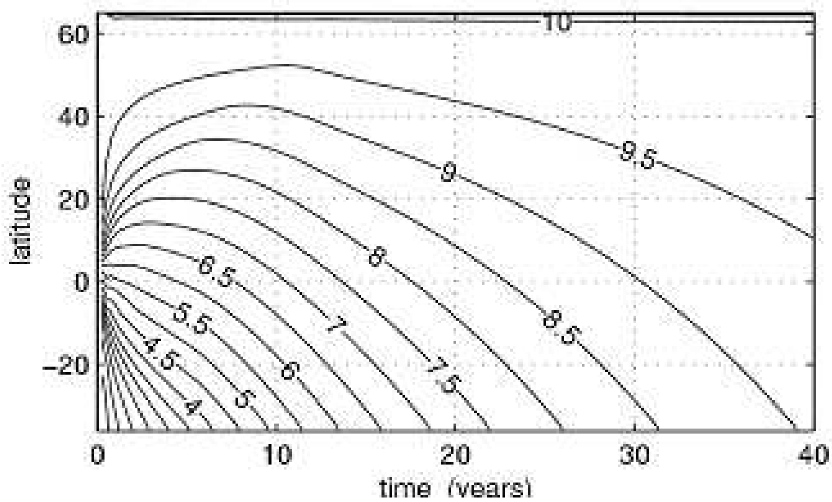
$$T(y, t) = T_N + \int_{y}^{y_N} c(y) \left[h_e - h_e\left(t - \frac{L}{c(y)}\right) \right]. \qquad (5.21)$$

Eastern boundary layer thickness, h_e (m):

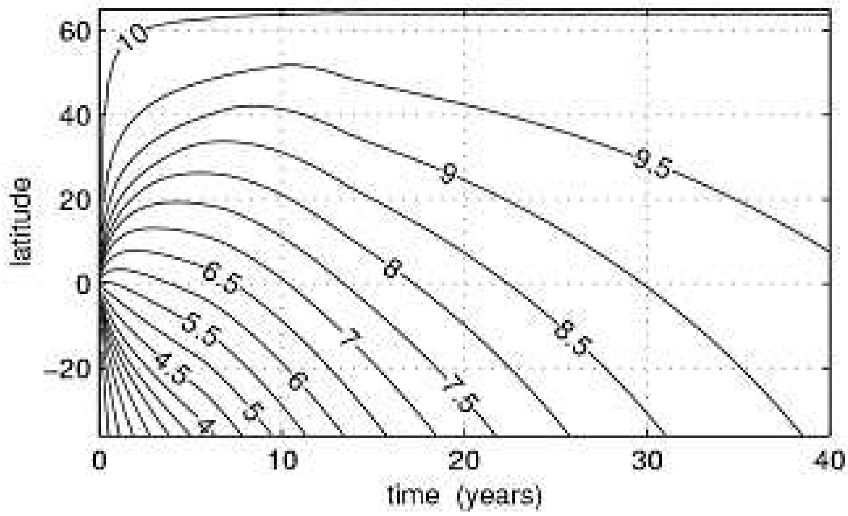


Northward transport (Sv):

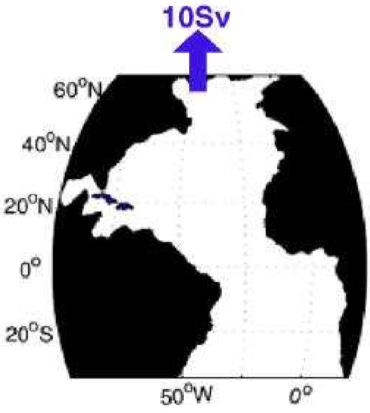
model



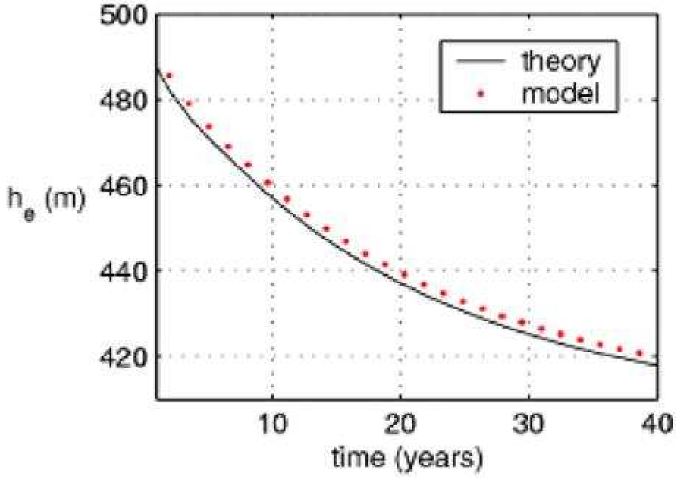
theory



Realistic Atlantic geometry

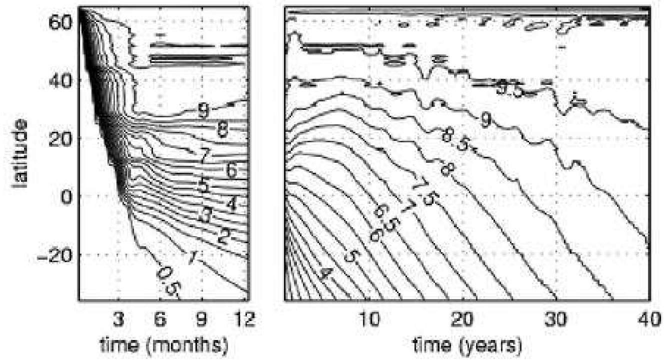


Eastern boundary layer thickness (m)

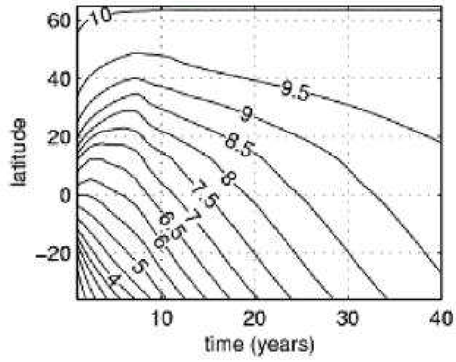


Northward transport (Sv)

Numerical model



Theory

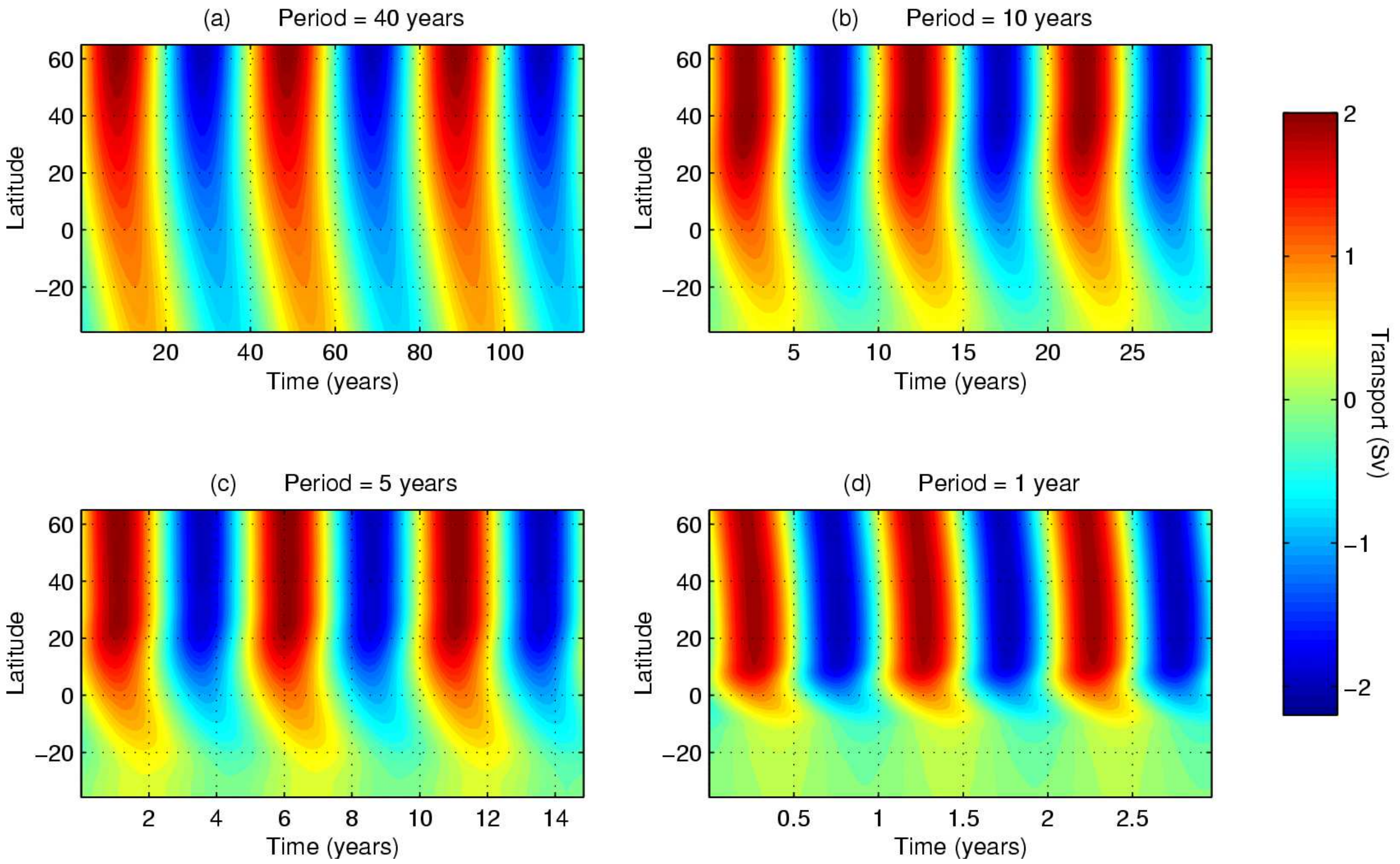


PERIODIC FORCING

Equatorial buffer $\Rightarrow$ the equator should act as a *low pass filter* to meridional transport anomalies.

Apply periodic forcing (Johnson and Marshall 2002b, 2004), $T_N = A \sin(\omega t)$:

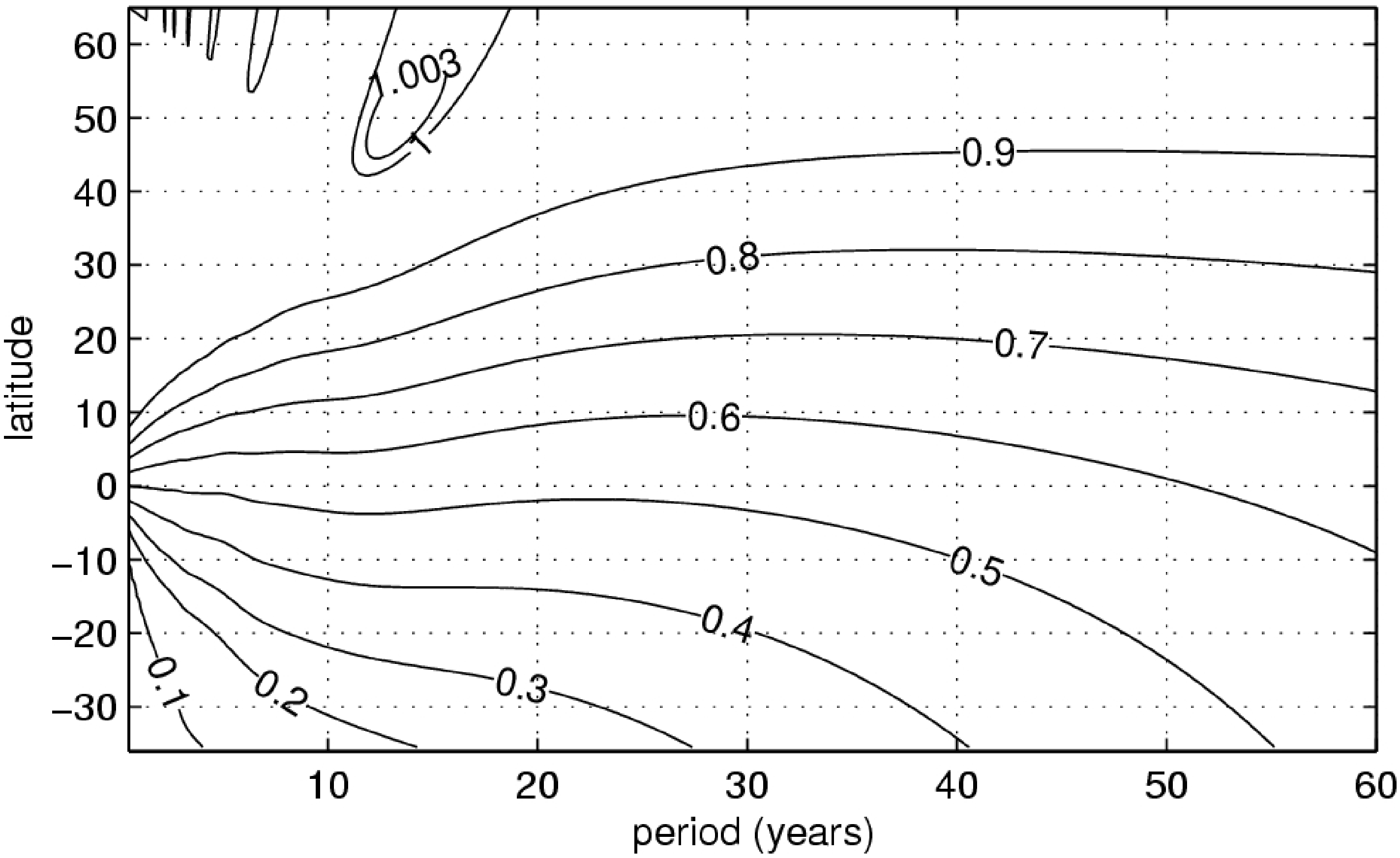
Northward transport (Sv):



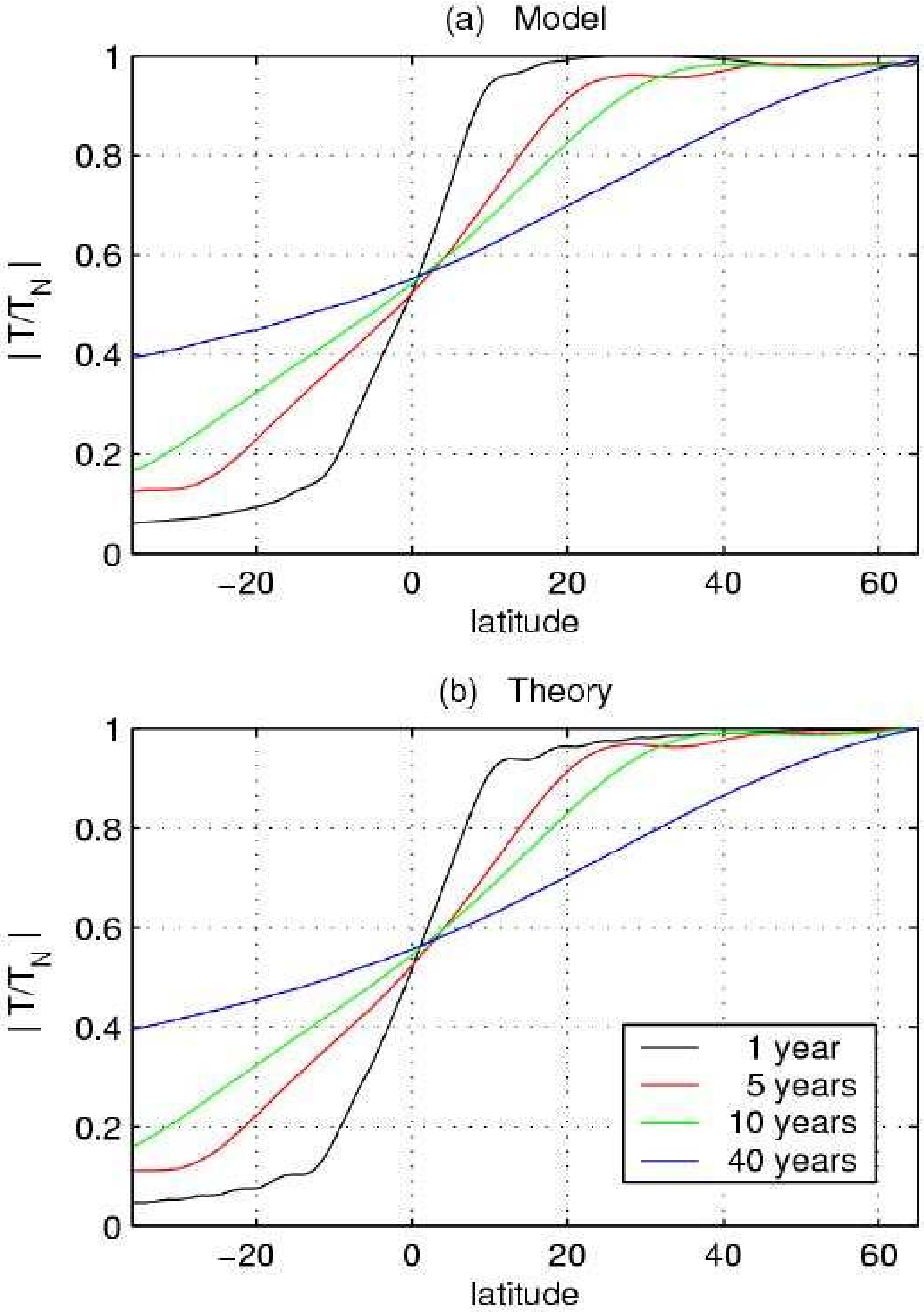
At high frequencies, the transport anomalies are confined to the hemispheric basin in which they are generated (and to the western margin of that basin).

Substituting $T_N = T_0 e^{i\omega t}$ into the delay equation (5.19) gives:

$$T(\omega, y) = T_N \frac{\left\{\frac{g'H}{f_S} - \int\limits_{y}^{y_N} c\left(1 - e^{-i\omega L/c}\right)\, dy\right\}}{\left\{\frac{g'H}{f_S} - \int\limits_{y_S}^{y_N} c\left(1 - e^{-i\omega L/c}\right)\, dy\right\}}. \tag{5.22}$$



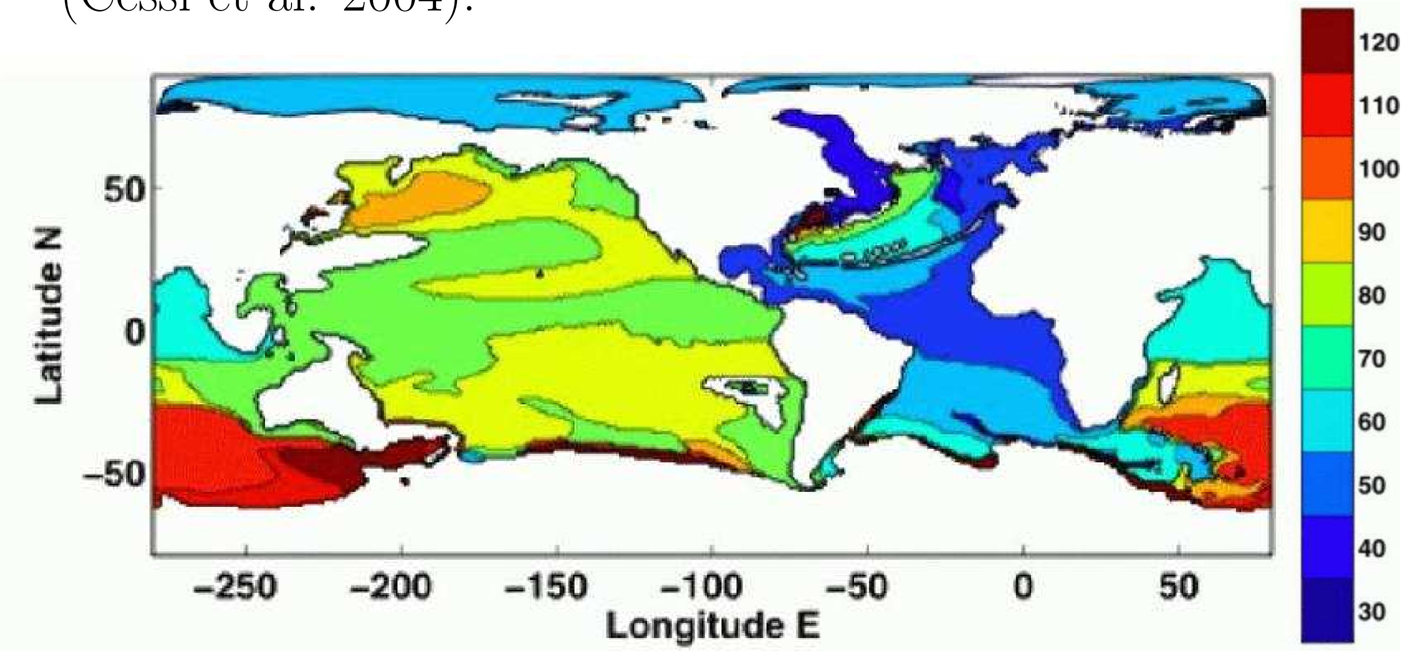
Note the weak resonance of *basin modes* adjacent to the northern boundary (see LaCasce 2000, Cessi and Primeau 2001, Cessi and Paparella 2001, Primeau 2002 for details of these basin modes.)


(a) Model
1
0.8
0.6
0.4
0.2
0
| T/T$_N$ |
−20
0
20
40
60
latitude
(b) Theory
1
0.8
0.6
0.4
0.2
0
| T/T$_N$ |
1 year
5 years
10 years
40 years
−20
0
20
40
60
latitude

ADJUSTMENT IN MORE COMPLETE MODELS

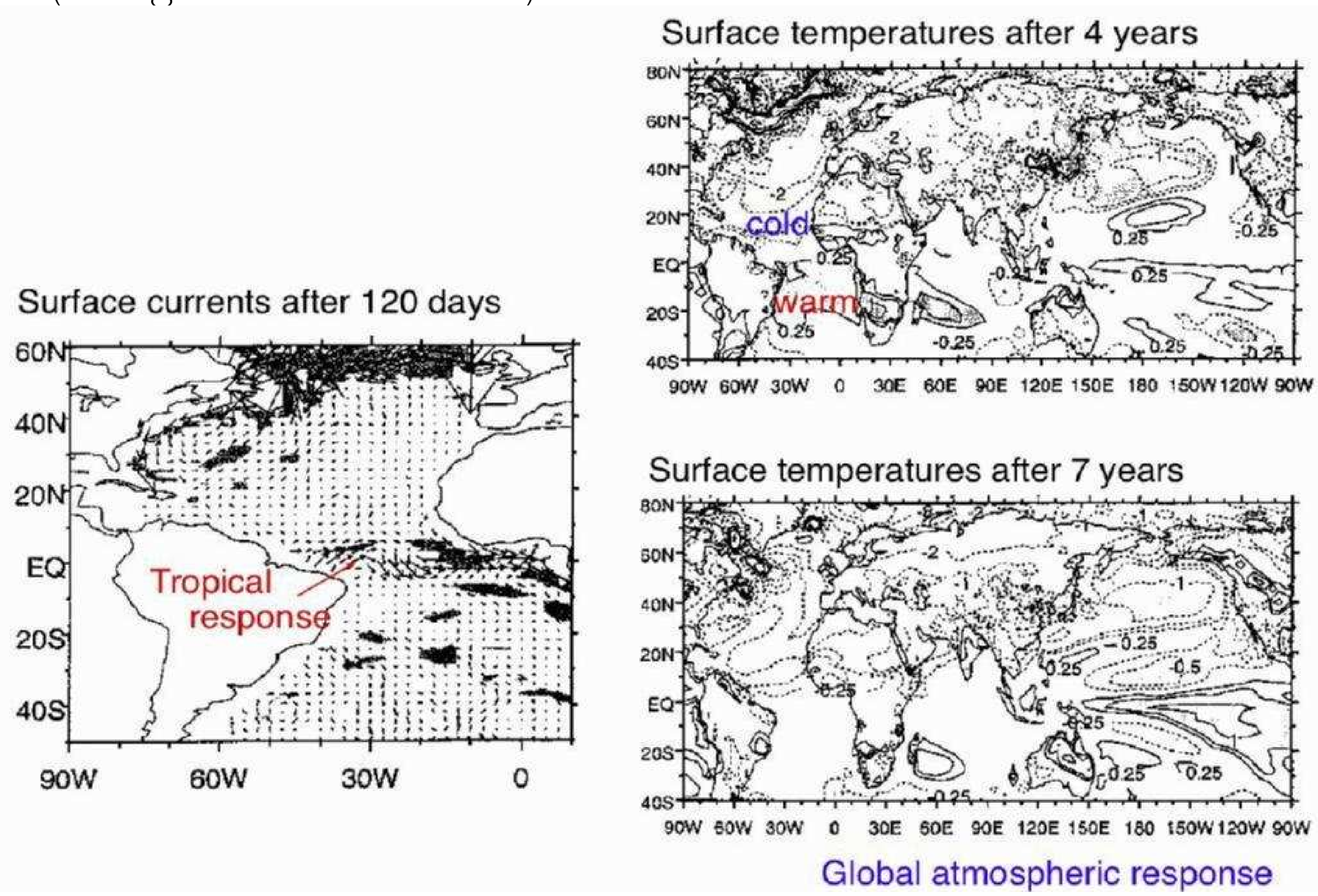
a. Ocean general circulation model

Time of arrival (years) of the first maximum in sea surface elevation in response to a sinusoidal freshwater flux applied to the Labrador Sea with a period of 100 years (Cessi et al. 2004):



b. Coupled ocean-atmosphere model

Response to a large instantaneous freshening of the high-latitude North Atlantic in HadCM3 (Dong and Sutton 2002):



Note the rapid response in the tropical Atlantic surface currents after only a few months. These currents in turn establish a tropical SST dipole anomaly.

This SST dipole subsequently impacts on the atmospheric intertropical convergence zone, thereby triggering a global atmospheric response.

SUMMARY OF KEY POINTS

- The ocean adjusts to localised forcing anomalies through the propagation of Rossby and Kelvin waves.
- On short time-scales, the response is largely confined to the hemispheric basin of the forcing through an equatorial buffer mechanism.
- The equator therefore acts as a low-pass filter to thermohaline variability.
- In a reduced-gravity ocean, the adjustment can be described through a simple analytical model for the eastern boundary layer thickness.
- Similar qualitative results are obtained in more complete models of the ocean and coupled ocean-atmosphere system.

REFERENCES FOR LECTURE 5

General reading

Anderson, D., and A. Gill, 1975: Spin-up of a stratified ocean with application to upwelling. *Deep Sea Res.*, **22**, 583-596.

Cushman-Roisin, B., 1994: *Introduction to Geophysical Fluid Dynamics.* Prentice-Hall.

Gill, A. E., 1982: *Atmosphere-Ocean Dynamics.* Academic Press.

Johnson, H. L., and D. P. Marshall, 2002a: On the response of the Atlantic to thermohaline variability. *J. Phys. Oceanogr.*, **32**, 1121-1132.

Siedler, G., J. Church, and J. Gould, 2001: *Ocean Circulation and Climate.* Academic Press.

Specific references

Cessi, P., K. Bryan, and R. Zhang, 2004: Global seiching of thermocline waters between the Atlantic and the Indian-Pacific ocean basins. *Geophys. Res. Let.*, **31**, L04302, doi:10.1029/2003GL019091.

Cessi, P. and S. Louazel, 2001: Decadal oceanic response to stochastic wind forcing. *J. Phys. Oceanogr.*, **31**, 3020-3029.

Cessi, P., and P. Otheguy, 2003: Oceanic teleconnections: Remote response to decadal wind forcing. *J. Phys. Oceanogr.*, **33**, 1604-1617.

Cessi, P., and F. Paparella, 2001: Excitations of basin modes by ocean-atmosphere coupling. *Geophys. Res. Let.*, **28**, 2437-2440.

Cessi, P., and F. Primeau, 2001: Dissipative selection of low-frequency modes in a reduced-gravity basin. *J. Phys. Oceanogr.*, **31**, 127-137.

Chelton, D. B., and M. G. Schlax, 1986: Global observations of oceanic Rossby waves. *Science*, **272**, 234-238.

Clarke, A. J., and C. Shi, 1991: Critical frequencies at ocean boundaries. *J. Geophys. Res.*, **96**, 10731-10738.

Dong, B.-W., and R. T. Sutton, 2002: Adjustment of the coupled ocean-atmosphere system to a sudden change in the thermohaline circulation. *Geophys. Res. Let.*, **29**, 1728.

Johnson, H. L., and D. P. Marshall, 2002b: Localization of abrupt change in the North Atlantic thermohaline circulation. *Geophys. Res. Let.*, **29**, 1083, doi: 10.1029/2001GL014140.

Johnson, H. L., and D. P. Marshall, 2004: Global teleconnections of meridional overturning circulation anomalies. *J. Phys. Oceanogr.*, **34**, 1702-1722.

Kawase, M., 1987: Establishment of deep ocean circulation dirven by deep-water production. *J. Phys. Oceanogr.*, **17**, 2294-2317.

LaCasce, J. H., 2000: Baroclinic Rossby waves in a square basin. *J. Phys. Oceanogr.*, **30**, 3161-3178.

Primeau, F., 2002: Long Rossby wave crossing time and the resonance of low-frequency basin modes. *J. Phys. Oceanogr.*, **32**, 2652-2665.

Wajsowicz, R. C., and A. E. Gill, 1986: Adjustment of the ocean under buoyancy forces. Part I: The role of Kelvin waves. *J. Phys. Oceanogr.*, **16**, 2097-2114.

Yang, J., 1999: A linkage between decadal climate variability in the Labrador Sea and the tropical Atlantic Ocean. *Geophys. Res. Let.*, **26**, 1023-1026.